Towards a unified bayesian geometric framework for template estimation in Computational Anatomy

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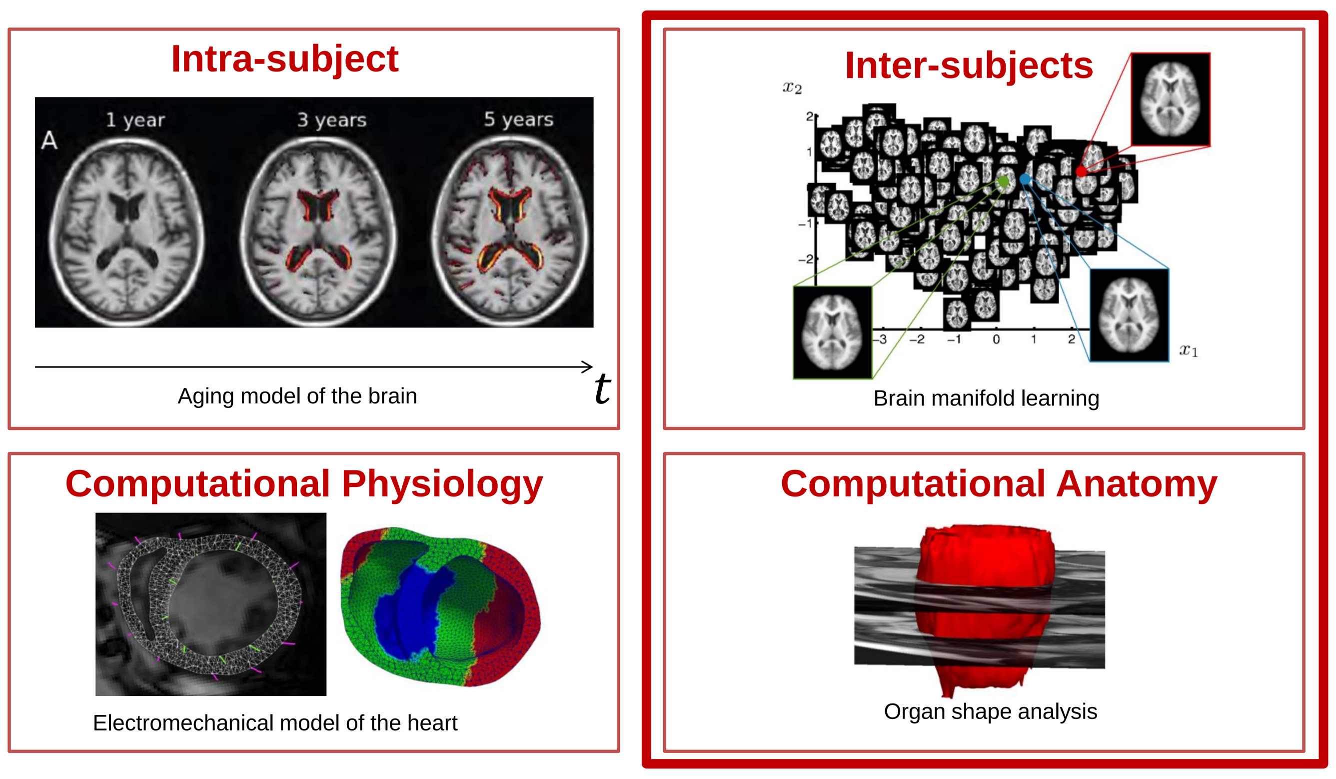
Computational Anatomy aims to model and analyze the variability of the human anatomy. Given a set of medical images of the same organ, the first step is the estimation of the mean organ's shape. This mean anatomical shape is called the template in Computer vision or Medical imaging. The estimation of a template/atlas is central because it represents the starting point for all further processing or analyses. In view of the medical applications, evaluating the quality of this statistical estimate is crucial. How does the estimated template behave for varying amount of data, for small and large level of noise? We present a geometric Bayesian framework which unifies two estimation problems that are usually considered distinct: the template estimation problem and manifold learning problem - here associated to estimating the template's orbit. We leverage this to evaluate the quality of the template estimator.

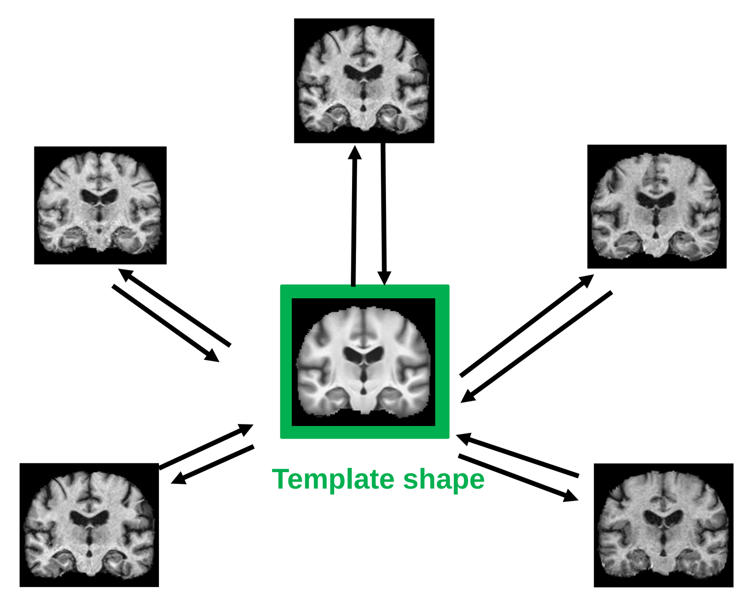
Template estimation in Computational Anatomy

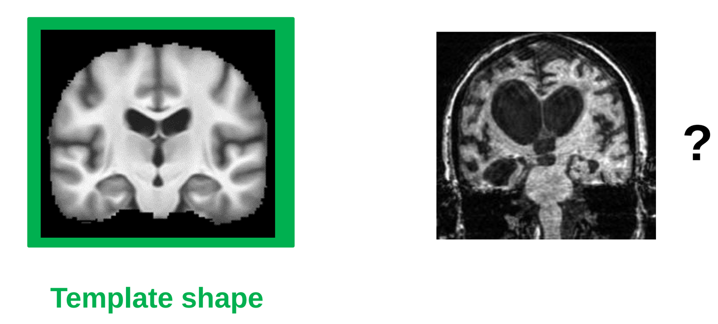
Computational Medicine relying on medical images



Images from: [Talbot and al 2013][Lorenzi and al, 2011][Gerber and al, 2010][Margeta and al, 2011]

First step: **template shape** computation → Second step: analysis

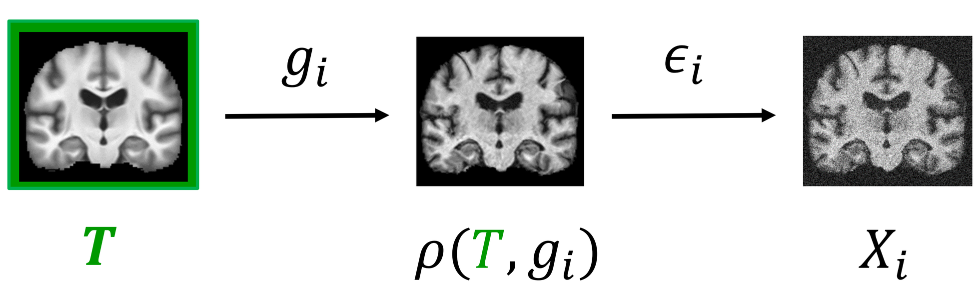




Template estimation as a non-linear model of Errors-in-Variables

Generative model of organs' shapes

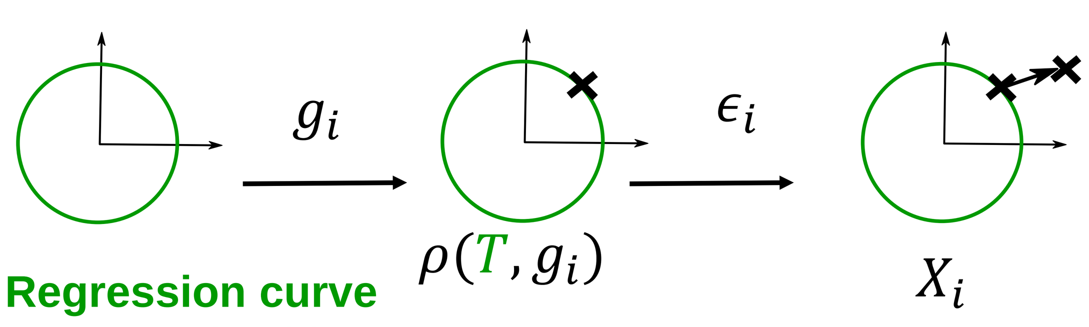
$X_i = \boldsymbol{\rho}(\boldsymbol{T}, \boldsymbol{g_i}) + \epsilon_i$
where $g_i \sim \mathcal{N}(g_0, \eta)$ i.i.d. and $\epsilon_i \sim \mathcal{N}(0, \sigma)$ i.i.d.



Goal:
Estimate the **template T**

Non-linear model of Errors-in-Variables

$X_i = \boldsymbol{\rho}(\boldsymbol{T}, \boldsymbol{g_i}) + \epsilon_i$
where $g_i \sim \mathcal{N}(g_0, \eta)$ i.i.d. and $\epsilon_i \sim \mathcal{N}(0, \sigma)$ i.i.d.

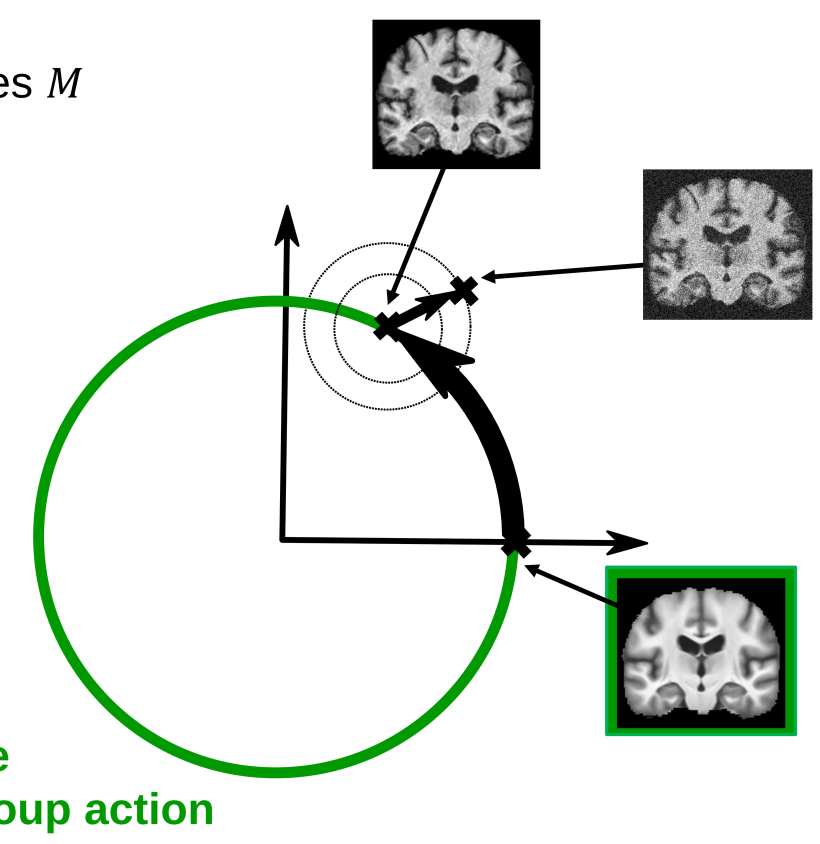


Goal:
Estimate the **curve parameterized by T**

Unification through Geometric Statistics

M: space of the images X_i's
G: Lie group of transformations
Action of G on M: $\rho: M \times G \to M$ denoted: $(X, g) \to \rho(X, g)$
Q: shape space, quotient of M by G

Space of images M



Different estimators of the template's shape

Functional model: g_i's are parameters

Likelihood: $L = \Pi_{i=1}^n \exp\left(-\frac{d_M^2(\rho(X_i, g_i), T)}{2\sigma^2}\right)$

- Modal approximation in (1): $\exp\left(-\frac{d_G^2(g, g_0)}{2\eta^2}\right) \simeq \delta_{g_0}$ i.e. $\eta \simeq 0$
- Adding regularization in (1): $+\frac{\sigma^2}{\eta^2} d_G^2(g_i, g_0)$

Maximum-Likelihood (MLE-F)

(1) $\forall i,\ \hat{g}_i = \text{argmin}_{g \in G}\ d_M^2(\rho(\hat{T}, g), X_i)$
(2) $\hat{T} = \text{argmin}_T \sum_{i=1}^n d_M^2(\rho(T, \hat{g}_i), X_i)$

$\hat{T} = \text{argmin}_T \sum_{i=1}^n \min_{g_i \in G} d_M^2(\rho(T, g_i), X_i)$
Frechet mean in the shape space

Structural model: g_i's are random variables

Likelihood: $L = \Pi_{i=1}^n \int_{g \in G} \exp\left(-\frac{d_M^2(\rho(X_i, g), T)}{2\sigma^2}\right) \exp\left(-\frac{d_G^2(g, g_0)}{2\eta^2}\right) dg$

Maximum-Likelihood: Expectation-Maximization algorithm (MLE-S)

(1) Expectation
(2) Maximization

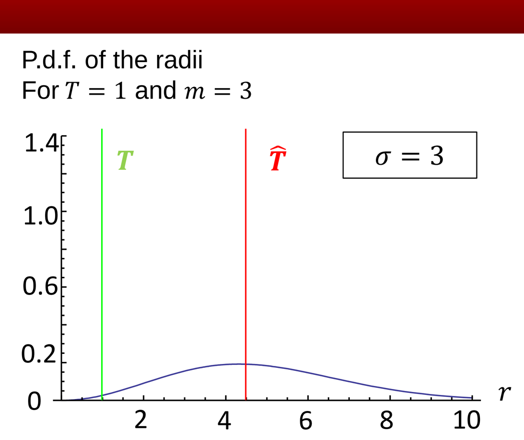
No closed form solution

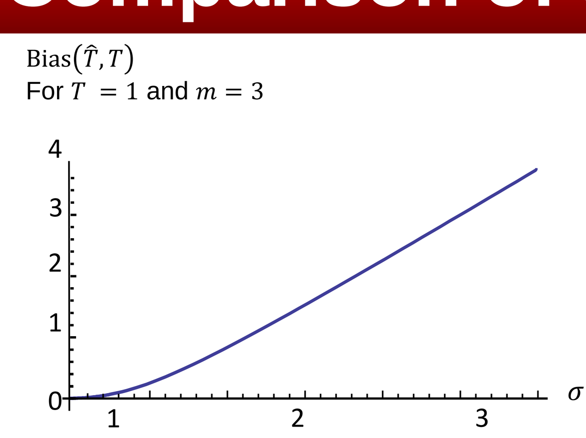
Adding priors: $p(T) = \text{cte}.\exp\left(-\frac{d_Q(T, T_0)^2}{2\sigma_T^2}\right)$ reweights metric in shape space; $p(g_0) = \text{cte}.\exp\left(-\frac{d_O(g_0, \overline{g_0})^2}{2\sigma_{g_0}^2}\right)$ reweights metric in the orbit; $p(\sigma) = \text{cte}.\left(\frac{\exp\left(-\frac{\bar{\sigma}^2}{2\sigma_\sigma^2}\right)}{2\sigma_\sigma^2}\right)^{a_\sigma}$; $p(\eta) = \text{cte}.\left(\frac{\exp\left(-\frac{\bar{\eta}^2}{2\sigma_\eta^2}\right)}{2\sigma_\eta^2}\right)^{a_\eta}$

Maximum-a-Posteriori (MAP-F)

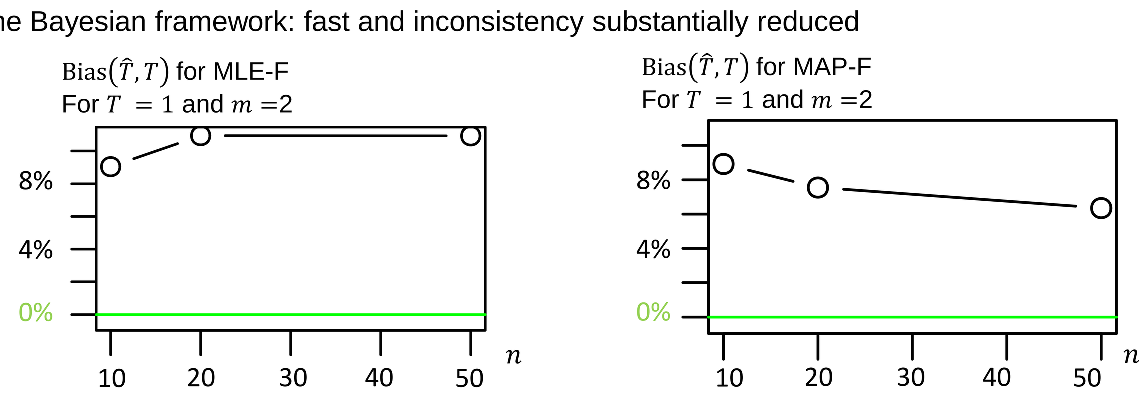
Comparison of the estimators

MLE-S: Consistent but slow
MLE-F: Fast but inconsistent





Improvement using the Bayesian framework: fast and inconsistency substantially reduced



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References: [1] Miolane, Holmes, Pennec. Biased estimators on quotient spaces (2015). [2] Allassonniere, S., Amit, Y., Trouve, A.: Towards a coherent statistical framework for dense deformable template estimation (2007). [3] Devilliers, Allassonniere, Pennec, Trouve. Frechet means top and quotient space might not be consistent: a case study (2015).